



Surname : Name :	ID : Instructor :	Signature	1	2	3	4	5	T
5 questions, 2 pages		Duration : 90 min	100 points					

Q1. Given that

$$f(x) = \begin{cases} e^{-1/x^2}, & \text{if } x \neq 0 \\ 0, & \text{otherwise} \end{cases}$$

find all critical and singular points of $f(x)$, and determining the intervals where $f(x)$ increases and where it decreases, find local extreme values of f .

Q2. Determine whether each of the statement below is true or false, providing short explanations if it is true and counter-examples if it is false:

(a) Every increasing function has inverse.

(b) If a function is continuous on an interval I , then it must be differentiable on I .

(c) If $f'(c) = 0$ for some point c in the domain of a function f , then $f(c)$ is either a local maximum or a local minimum value of f .

(d) If both $f'(x)$ and $f''(x)$ exists, with $f''(x) \neq 0$ and $f'(x)/f''(x) > 0$ on the open interval (a, b) , then $f(x)$ must be either increasing or decreasing on (a, b) .

Q3. Compute the following limits.

(a) $\lim_{t \rightarrow \frac{\pi}{2}} \frac{|\cos t - \sin t|}{|\cos t| - |\sin t|}$

(b) $\lim_{x \rightarrow \infty} x^{\sin(\frac{1}{x})}$

Q4. Find the point(s) on the curve $2 \tan^{-1}(y/x) = \ln(x^2 + y^2)$ at which the tangent line is horizontal.

Q5. A point $\mathbf{P}$ in the first quadrant of the Cartesian plane is moving along a circle centered at the origin. As the point $\mathbf{P}$ moves, the x-intercept of the tangent to the circle at $\mathbf{P}$ moves on the x-axis at the rate of $\mathbf{1}$ unit per second.

At what rate is the point $\mathbf{P}$ moving when the tangent line has slope $-\mathbf{1}$?